Saturday January 10, 2015

8:00 a.m.  Project NExT Workshop  Rm 217D, Convention Ctr

8:00 a.m. Hansun To*, Worcester State University  Rm 209, Convention Ctr

*IBL in a Liberal Arts Mathematics Course.*

Each semester Worcester State University offers several sections of Survey of Math, MA105. This is a course taken by many undergraduates not majoring in the mathematical or life sciences, to satisfy a quantitative reasoning course of Liberal Arts and Science Curriculum requirements for graduation. It is intended to support students' liberal arts and social science interests, by investigating applications of mathematics in contexts which are relevant to individuals without strong interests in the mathematics. I have adapted the inquiry based lecture/learning to this course since 2012. I will share the improvement of assessment, compare with other non-IBL sections and pros and cons of using this particular technique at the introductory level of studying.

8:20 a.m. Brian Johnson, Katie Johnson, Florida Gulf Coast Univ.  Rm 209, Convention Ctr

*Experiences with Process Oriented Guided Inquiry Learning (POGIL) in a general education mathematics course.*

We will highlight some of the activities used in a general education math course. Finite Math focuses on problem solving and mathematical reasoning through studying concepts in geometry, set theory, logic, probability, and statistics. Eighteen 75-minute activities were developed and facilitated in four sections of 35 students each over two semesters. The activities were designed specifically for use in a POGIL (Process Oriented Guided Inquiry Learning) classroom. Feedback from students was exceedingly positive, failure rates decreased dramatically, and we will share other anecdotes.

8:40 a.m. Victor I Piercey*, Ferris State University  Rm 209, Convention Ctr

*Inquiry-Based Learning in a Quantitative Reasoning Course for Business Students.*

"Just tell me how to solve the problem!" Sound familiar? Promoting buy-in to inquiry-based learning is particularly challenging for college freshmen who are not mathematics majors. In this talk, I will share my approach to inquiry in a quantitative reasoning course for business students. In particular, I will address the interaction between curricular materials and inquiry-based pedagogy, my strategies for promoting and sustaining student buy-in, and how I scaffold the level of inquiry throughout the course. In addition, I will share data concerning the impact on math anxiety and beliefs.

9:00 a.m. Li-An Daniel Wang*, Trinity College  Rm 209, Convention Ctr

*Modified Moore Method in Introduction to Proofs.*

Even for students having taken a course introducing them to proofs, real analysis can still be very challenging. We designed a course that exposes students to more proofs and basic analysis ideas, using a modified Moore Method. We discuss how the course was conceived and received by the students.
MAA Minicourse #3: Part A Introduction to process-oriented, guided-inquiry learning (POGIL) in mathematics courses

9:20 a.m. McKenzie R. Lamb*, Ripon College  Rm 209, Convention Ctr

Using IBL to Bridge the Gap Between Math for Liberal Arts and Intro to Proofs.

Teaching a course that is supposed to be simultaneously an introduction to proofs for math majors and a math for liberal arts course for non-majors is a daunting undertaking. The primary challenge is in keeping the non-majors on board while keeping the majors from getting bored. I will describe using a hybrid IBL structure to meet this challenge in a course at a small, liberal arts college. I will focus on the nuts and bolts of implementation: grading, presentations, etc.

9:40 a.m. Kevin Hartshorn*, Moravian College.  Rm 209, Convention Ctr

Writing Across the Curriculum and IBL.

Writing across the curriculum (WAC) programs are institution-wide initiatives designed to encourage students to use writing not just as a means of communication, but as part of the learning process itself. Through journals, writing workshops, reading response essays, and other targeted writing assignments, students are called to engage in the learning process. In this presentation, I will share ways in which my work with our first year seminar and WAC has translated to stronger inquiry-based lessons in mathematics. While my talk will focus on activities in first semester calculus and basic liberal-arts math courses, I have found that many of these techniques have fared quite well in my junior/senior algebra and geometry courses.

10:00 a.m. Paul E Seeburger*, Monroe Community College  Rm 209, Convention Ctr

Exploring Velocity and Acceleration Vectors Visually.

In multivariable calculus, we ask students to calculate vector-valued functions for velocity and acceleration, given a position function. Students often find it easy to visualize the velocity vector being tangent to the space curve, but they rarely have a clear picture of the acceleration vector and its relationship to the motion and to the corresponding velocity vector. Using a freely available online multivariable calculus applet called CalcPlot3D, students can complete a guided exploration of velocity and acceleration. As part of this guided activity, students complete a pre-test, answer exploration questions, and then complete a post-test. After students have completed this activity, there is often a lively class discussion about the interaction between the acceleration and velocity vectors they observed in the dynamic examples from the exploration. Through this discussion most misunderstandings are cleared up, and students become more confident in what they learned from the exploration. In addition to demonstrating this online exploration, analysis of the pre- and post-test results and student comments on their own learning will be shared. CalcPlot3D is part of an NSF-funded grant project (DUE- CCLI

10:20 a.m. Amy Ksir*, United States Naval Academy  Rm 209, Convention Ctr

Inquiry-Based Calculus III.

The first author has taught Calculus III (multivariable calculus) nine times in the past 11 years. Two years ago she took the plunge and switched from an interactive lecture / homework-from-the-textbook format to an inquiry-based format centering on student presentations. We will present a comparison of various student learning outcomes from before and after, analyzed with the help of the third author. We will describe the problem set developed by the first and second authors that replaces the course textbook; it has also been successfully used by the second author in an IBL class centered on group work.
Raising Calculus to the Surface: Using Physical Surfaces to Facilitate Inquiry-Based Learning in Multivariable Calculus.

One significant challenge in bringing inquiry-based learning materials to multivariable calculus is creating opportunities for students to engage meaningfully with functions of more than one variable. To meet this challenge, we have developed a set of physical surfaces, measurement tools, and corresponding activities that allow students to discover many of the key concepts from multivariable calculus. In this talk we will give an overview of this project, demonstrate how these materials can be used to develop methods for optimizing along a constraint, and share results from students and instructors implementing these materials in their classroom. (This research is funded in part by the National Science Foundation as part of the Raising Calculus to the Surface project, DUE #1246094)

Cryptology By Discovery: Favorite Inquiry-Based Activities.

At the University of Redlands, we offer a one-month intensive term course as our transition to upper level mathematics course. While this course has been offered with a variety of topics, students have responded especially well to using topics from cryptology. One of the most important aspects of making this course successful is incorporating a wide variety of discovery-based activities. In this session I will present some of the most successful activities that I have found/developed for engaging students with cryptology. While this course is designed for first and second year mathematics and computer science majors and minors, recommendations for how these materials could be adapted for other levels will also be included. A link to a website with sample handouts and interactive demos (via embedded Sage cells) will be provided.

An inquiry-based learning in Developmental Mathematics Course.

Mathematics teaching and learning can be seen, through a cultural lens, as an exchange of information. The richness of that exchange, and in turn the culture of the mathematics classroom, depends upon the way in which inquiry is employed. This presentation describes the implementation of a rigorously inquiry-based approach to mathematics education in the context of a developmental-level mathematics course. over the first two years of this implementation, students enrolled in inquiry-based learning developmental mathematics sections were significantly more likely to pass the course than students taking the traditional developmental course. Furthermore, of the students who went on to take credit-bearing mathematics, the students from the inquiry-based sections were more likely to pass that course than the students from the traditional developmental course.
Inquiry-Based Activities in a Precalculus with Trigonometry Course.

I have been teaching upper-level courses via a modified Moore Method for several years now, but did not feel this approach would be as suitable for our freshman-level Precalculus with Trigonometry course. However, the underlying principles of the Moore Method have found their way into my pedagogy in this setting through in-class activities, group work, and student sharing of solutions. I will share some of these activities and describe the logistics of running the course in this fashion, and will close with some outcomes, both measured and anecdotal.

Inquiry-Based Learning on the Way to Calculus.

Studies that we did at UAB in 2010, 2011, and 2013 point, in different ways, to the potential for IBL class meetings in pre-calculus courses to improve the chances of students to perform satisfactorily in Calculus I. The studies in 2010-11 were quasi-experimental studies of incorporating IBL/Group Learning sessions in Basic Algebra (a remedial course), reported at the Conferences on Research in Undergraduate Mathematics Education the subsequent years. The 2010-11 studies led to a change in how we teach Basic Algebra from 2012 onward. The 2013 study was a statistical study of success of students in Calculus I in the period 2006-2012 based upon the first mathematics course taken at UAB. The 2013 study pin-pointed where we could get the most 'bang for the buck' in subsequent student success in Calculus, if we made an appropriate change in instruction. Of course, the study does not imply what type of change is appropriate. I will outline a two-pronged approach (one quasi-experimental, one statistical) to help resolve this issue.

IBL College Algebra.

For those of us who employ inquiry in our learning, it seems natural to employ IBL in teaching. However, college algebra students may be new to IBL and have difficulties adapting to this approach to learning. In acknowledgement of the importance of college algebra as the final course in mathematics for many students and the difficulties some of them have with the IBL method, the presenter identified some possible changes to improve his delivery of an IBL college algebra course. This report discusses effectiveness of changes that began in 2011 with support from The Educational Advancement Foundation.

A Modified Moore Method in Pre-calculus: Achievement, Attitudes, and Beliefs.

The presenters have completed a two-year quasi-experimental study of the use of a Modified Moore Method (MMM) in Pre-calculus. This study included six traditional lecture classes and seven MMM classes taught by three instructors. Both quantitative and qualitative data was analyzed to investigate achievement as well as attitudes and beliefs. In this talk, we will present a brief summary of our results with a particular focus on an 'Openness to Inquiry Based Learning' survey that we have developed for assessing students' preferences for Student-Centered or Teacher-Centered instruction. Students who score higher on this survey, tended to perform better on the final exam; indicating that a key element to success in an MMM course may be the students willingness to participate.
**Reasons behind rules -- aligning the ‘unreachable’ asymptotes.**

During my research on student conceptions of different calculus and pre-calculus concepts, calculus students accidentally discovered that asymptotes are geometric constructs that both functions and non-functions can hold. These students came from diverse academic standings in mathematics while they held the commonality of choosing their careers in the engineering track. The students were marveled by their own discovery, they took pride that they explored the concepts by actively listening to their peers, pointing out the discrepancies in each other's views, looking at different examples and negotiating meanings that aligned better with the mathematical truth accepted by the mathematics community. In the end, they also cleared their long-standing confusion on what fits and does not fit with characteristics of asymptotes. What I learned from this experience as an instructor was that well-crafted collaborative assignments could help build confidence and enthusiasm in mathematics students. During this presentation I will discuss the details of this episode to demonstrate what collective, discovery learning efforts could accomplish.

10:00 a.m.  **Kathleen Grace Kennedy*, Wentworth Inst of Technology**  
**Naomi Ridge**, Wentworth Institute of Technology  
**Rachel Maitra**, Wentworth Institute of Technology  
**James O’Brien**, Wentworth Institute of Technology  
**Franz Rueckert**, Wentworth Institute of Technology  

**Teaching Physics-Calculus with Applications to Engineering.**

At Wentworth Institute of Technology, several engineering majors take physics I and calculus I at the same time. In the past, these courses being out of synch have caused difficulties for biomedical and mechanical engineering students taking them as co-requisites. In our presentation, we will report on the materials we developed and preliminary results on the impact of our course. Our team developed curriculum for a synergetic inquiry-based course integrating physics and calculus, such that each subject reinforces the other. Students obtain a visual or physical ‘picture’ to support the comprehension of calculus; likewise, physics will be presented not merely as an application of calculus but as its raison d’etre. In summary, we are teaching calculus through the lens of physics using context-driven and inquiry-based course material. Our work was funded by a grant from the Academy of Inquiry Based Learning and the Education Advancement Foundation.

10:20 a.m.  **Therese Shelton*, Southwestern University**  

**POGIL Flu for Calculus: Influenza Data to Help Students Investigate Antiderivatives, Accumulations, and FTC.**

Student responses about accumulation functions and The Fundamental Theorem of Calculus improved after completing an IBL-based activity using u data. The activity was designed according to guidelines from an MAA PREP POGIL Workshop.

10:40 a.m.  **Silvia Saccon*, The University of Texas at Dallas**  

**Engaging calculus students through problem-solving workshops.**

To foster students' active engagement in their learning and to promote their conceptual understanding of calculus, I started to run my classes as problem-solving workshops. Students experience a full immersion in problem-solving activities by collaborating in small teams at the board on problems designed to build and stretch their conceptual understanding of calculus. Deep engagement in mathematical tasks, combined with immediate feedback through discussions with peers and me, helps students develop their critical reasoning, problem-solving, and communication skills. In this talk, I will describe class structure and activities performed by students, discuss benefits and challenges in this learner-centered environment, and review student feedback and the impact of this approach on student learning.
**To δε or not to δε.**

What is the role of the definition of a limit in an introductory Calculus course? This definition can overwhelm young students, and yet without it these same students can become frustrated that there is no way to provide solid answers to questions about Calculus. In this talk, I will describe a task that facilitates guided-reinvention of the definition of limit adapted from the work of Michael Oehrtman and other RUME scholars. I will put the task in the context of a course that uses notes by Brian Loft and connect it to the larger inquiry trajectory in these notes.

**The Development and Implementation of Inquiry-Based Learning Projects in Precalculus and Calculus.**

This paper discusses the evolution of the inquiry-based learning projects used in our courses throughout the 2013-2014 school year: Precalculus I (Building a Bridge), Precalculus II (Exploring Trigonometry through Sound) and Calculus I (Formula 1 Racing Strategy). We will include the description, technology and implementation methods used for each project along with student feedback.

**Effective implementations of POGIL in the Calculus I classroom.**

Process Oriented Guided Inquiry Learning (POGIL) is an instructional strategy that guides students toward the discovery of a particular concept by working in self-managed teams of three or four on specially designed POGIL activities. A POGIL classroom focuses both on the achievement of content knowledge and the development of process skills such as effective communication of mathematical arguments, information processing, critical thinking, teamwork, and metacognition. In this talk we will give examples of POGIL activities used in Calculus I and report data on the effectiveness of POGIL implementation at a variety of institutions.

**Inquiry-Based Learning in Honors Calculus I.**

First year Calculus courses are usually taught with the purpose of teaching students a set of specific methods and procedures of calculus related to limits and derivatives, and their applications to “real life” problems. While this may be an efficient way to convey abundant information to students in a limited time, it frequently results in students’ misunderstandings of basic concepts of calculus and a limited ability to prove and present arguments in mathematics. In this paper we discuss the implementation of an inquiry-based Calculus I course for honors students. Through the examination of assessment data, survey data, and anecdotal evidence we show that the course fostered positive attitudes about mathematics and encouraged multiple approaches to solving problems. We show that this type of course has some clear advantages over lecture-based courses. We also discuss the challenges of implementing such a course into a class where there is an expectation of covering a considerable amount of material.
Yun Lu*, Kutztown University     Rm 209, Convention Ctr

Applying the Inquiry-Based Learning Elements in Teaching Calculus II class.

During this talk, I want to share my experience of applying the inquiry-based elements to enhance students' learning in my calculus class. I will talk about my motivation, problems encountered, success received, as well as students' feedback.

Milos Savic*, University of Oklahoma    Rm 209, Convention Ctr

Incorporating Social Norms and "Leveling Up" to a Medium-Sized Calculus II Course.

Social norms are roughly defined as the 'rules of the environment' of the classroom, with the rules either verbally or non-verbally communicated. I aimed to incorporate two main ideas in my Calculus II course: one can level up" (borrowing a video game phrase) to reach benefits and ultimately a grade in the course, and that the environment is one that is non-judgmental, meaning students cannot judge each other for what they state verbally. This has resulted in many students participating frequently in this Calculus II IBL course (of 35 students), which has, in turn, caused discussion to be fruitful. I elaborate on both ideas in my presentation, noting that the content of the course is non-essential: one can use the two ideas in other courses, specifically in first- and second-year courses.

Uri Treisman*, University of Texas at Austin             Rm 217D, Convention Ctr

Project NExT Lecture
Ruminations on learning to teach: developing pedagogical intimacy, productive persistence, and other aspects of critical pedagogy.

MAA Poster Session on Projects Supported by the NSF Division of Undergraduate Education

Stan Yoshinobu*, California Polytechnic State University, San Luis Obispo
Carol Schumacher, Kenyon College
Matthew Jones, California State University, Dominguez Hills
Sandra Laursen, University of Colorado

Supporting Pedagogical Innovation for a Generation of Transformation via Inquiry-Based Learning in Mathematics (SPIGOT).

Dianna Spence*, University of North Georgia
Brad Bailey, University of North Georgia

Discovery Learning Projects in Introductory Statistics.

James A. Mendoza Epperson*, University of Texas-Arlington
Julie Skinner Sutton, University of Texas-Arlington
Lynn Peterson, University of Texas-Arlington
Ramon Lopez, University of Texas-Arlington
Kevin Schug, University of Texas-Arlington
Carter Tiernan, University of Texas-Arlington

The Arlington Undergraduate Research-based Achievement for STEM (AURAS).
Developing a set of IBL course notes for integral calculus: ideas, challenges, and a request for suggestions.

There are unique challenges using IBL techniques in a calculus classroom: pressure to cover enough material to prepare students for another course, more students not majoring in mathematics, and large class sizes. These challenges (among others) extend to the development of IBL course notes. While the author has had some success with the creation of a set of notes for differential calculus, another set for integral calculus is proving to be more difficult. A preliminary outline of a set of notes for integral calculus will be discussed, along with the challenges that have been encountered. Suggestions for solutions to these challenges will be solicited.

Teaching Calculus 1 and 2 using Inquiry.

We will present an example of how Calculus 1 and 2 can be taught without any lecture, using inquiry-based techniques including small group work and large class discussion. The materials (see www.iblcalculus.com) were developed by Mairead Greene and Paula Shorter at Rockhurst University, MO, and have been used several times at Westfield State University, MA. The goal of the materials is for the students to develop a deep conceptual understanding by engaging them in the process of independently reasoning from concepts and prior knowledge to answer questions, solve problems and develop definitions and theorems. Data and anecdotes provide evidence of the success of this approach.

Inquiry-based learning of transcendental functions in calculus I and II.

Developing a deep understanding of transcendental functions such as ex and sin x should be an essential secondary goal of any calculus sequence. In recent semesters, we have pioneered inquiry-based calculus learning activities engaging students in building a series of polynomials with self-similar derivative structures. While using educational technologies in their explorations, students in peer groups develop their understanding through algebraic, numerical and geometric representations. This discourse facilitates students' construction of the series representations of transcendental functions. Students discover that the problem of their synthesis can be reduced into a sign pattern problem ultimately solved by devising a number whose powers reproduces the pattern. First-semester work culminates in the discovery of the Euler's Formula and paves the way for second-semester applications from the integral standpoint. We will further share our philosophy, techniques and results.
Small-group activities instead of examples: an inquiry-based approach to calculus.

Historically, calculus texts and calculus instruction have presented students a large number of completed examples. Often, students then solve related problems by emulating work done in the examples. In an effort to instead have students build their own intuition, think more independently, and discover key ideas themselves, I have developed a large (and freely available) collection of activities that engage students in learning calculus in a more inquiry-based style. In this talk, I will share some sample activities, discuss the structure of a typical class meeting, and reflect upon student feedback and outcomes with this approach.

IBL Course Notes for Calculus I, II, & III.

Calculus can be a fertile recruiting ground for majors. When teaching calculus early in my career, I would see students who sought a deeper understanding perform poorly on rote exams. Potential mathematicians were being lost because I was rewarding rote performance over mathematical inquiry! This talk addresses a set of calculus notes that engages students from the first day by having them solve problems and discuss solutions with the class. Started as an NSF project fourteen years ago, these notes have been used in one form or another at dozens of schools including: Cal Lutheran, College of Idaho, Drury, Holy Cross, Lamar, Lewis & Clark, NAU, SUNY Potsdam, and USNA. These notes are freely available, problem-based, include practice sets and cover all of Calculus I-III.

Interactive Engagement in Calculus Labs at Missouri S&T.

Over the course of the past year the Department of Mathematics and Statistics at Missouri University of Science and Technology has undertaken a redesign of our three semester calculus sequence. During the Fall 2014 semester we pilot tested elements of this redesign in one section of our Calculus I course with an enrollment of approximately 120 students. One of the components of the redesign is to dramatically increase student engagement during their lab (recitation) section through the use of inquiry-based techniques. This talk will provide an overview of observations and results from the pilot, as well as future plans.

Daily Student Presentations in Quantitative Reasoning and Calculus.

Inquiry-based learning is becoming more and more common in upper-level proofs-based courses. One popular model involves students presenting homework problems at the board while their classmates ask questions, provide suggestions for improvement, and take notes. While this is a useful model for proofs-based courses, can it be beneficial in freshman math courses as well? I firmly believe so. In the last year and a half, I have incorporated daily student presentations into my Quantitative Reasoning courses as well as Calculus I and II, with encouraging results. I will share the details of my methods as well as my reasons for believing that this model promotes student learning.
Planar Hyperbolic Geometry through Inquiry.

In this talk, we will describe an undergraduate course on Hyperbolic Geometry in the plane. This course is intended for undergraduate Math Majors who may NOT necessarily have had much practice in and exposure to mathematical proofs. The author has been teaching the class in this current version for 3 years (every fall). The course is taught with a focus on Inquiry. In a first phase, students are encouraged to make their own discoveries and conjectures through activities and problems. In a second phase, the class formalizes the results which are then proved by the students themselves in class and in assignments. A sample of activities and problems from this class will be shared and discussed during this session.

Monday January 12, 2015

8:00 a.m.  C Ray Rosentrater*, Westmont College Rm 204A, Convention Ctr

An IBL-influenced Approach to Teaching Linear Algebra.

There are many flavors of inquiry based learning. This talk will present my experience in a spring 2013 offering of linear algebra that eliminated most lectures but provided more support and direction than the traditional Moore method. I will present examples of the prompts used in developing the theory of projections and will conclude with some reflections on the relative success of this approach in comparison to previous lecture-heavy offerings.

8:00 a.m.  William G. Hager*, Texas Lutheran University Rm 209, Convention Ctr

Teaching an Inquiry-Based Elementary Linear Algebra Course at a Small Liberal Arts University.

The Elementary Linear Algebra course at Texas Lutheran University is a hybrid inquiry-based/lecture course usually offered once a year. The small class size and variety of majors in the classroom presents challenges for inquiry-based teaching. In this talk, we will detail some of these challenges and brieﬂy discuss the set of notes used for the course. We will also look at data from final exams, student surveys, and student comments.

8:20 a.m.  Theron J Hitchman*, University of Northern Iowa Rm 209, Convention Ctr

IBL Linear Algebra with a mixed audience and Sage.

We present a framing of an IBL classroom aimed at a mixed audience of students from mathematics and from client departments that has a signiﬁcant technology component. The main adaptations involve ﬁnding a reasonable standard of argument for a course that is not based on formal proof, and developing a way to integrate the use of the mathematical computing software Sage as a tool for mathematical investigation.
Creating and Sustaining Productive Whole Class Discussions.

In order to create a learning environment that encourages students to ask good questions, an instructor needs to initiate, sustain, and facilitate substantive and rigorous whole class discussions. However, creating such inquiry-oriented classrooms is no easy task. In this presentation I draw on insights from research projects in linear algebra and differential equations to highlight five goals for student participation and corresponding instructor discourse moves that can be used to achieve these goals. The five goals for student participation are: (1) helping individual students share their own thoughts, (2) helping students orient to and listen carefully to one another, (3) helping students deepen their reasoning, (4) helping students engage with others’ reasoning, and (5) building on and extending students’ ideas. For each of these goals, prototypical instructor prompts, questions, and requests will be tendered provided using examples from research projects in linear algebra and differential equations. These prototypical instructor moves constitute a set of routines that others can adapt in their efforts to create inquiry-oriented classrooms, whether the content be linear algebra or abstract algebra or real analysis.

Computational inquiry in elementary statistics.

The profusion of data in almost every aspect of applied science has made statistics an important course in the undergraduate curriculum. Unfortunately, many undergraduates who take elementary statistics have an uneasy relation with mathematics, and strongly theory-based approach can lead to frustration and other negative reactions. On the other hand, introductory statistics courses often have a tightly prescribed set of topics, coverage of which can be hard to reconcile with a more open ended, exploratory approach. The purpose of this talk is to report on my experience using computational guided inquiry in the R programming language as a means of bringing an inquiry based learning approach into the elementary statistics classroom. The computational work is structured just like the sequenced activities of conventional inquiry based learning, but it is implemented on the student’s own laptops, and is augmented by group work, file sharing, and physical lab notebooks. In additional to building fluency with statistical ideas, this approach also cultivates computational literacy, and ultimately empowers students to view their laptops as exploratory tools.

Exploration and Inquiry in an Introductory Course for Mathematics Majors.

My department is offering an introductory course for Mathematics majors, modeling Mathematics as a laboratory science. The course, based on material developed at Mount Holyoke College, requires only pre-calculus and co-enrollment in Calculus I. Students in small groups explore several rich mathematical topics on their own. They perform mathematical experiments (with the help of a computer algebra system), formulate, test and refine conjectures, and finally try to prove their conjectures. At the end of each two week laboratory, the student teams write up their findings in a laboratory report.
What we say/What they hear: Culture Shock in the Classroom.

In addition to learning specific mathematical content, our students need to learn to think like mathematicians. Indeed, helping them to do this should be at the center of every class we teach. Our students must become immersed in a kind of mathematical “sub-culture” in which they develop skills and practices that allow them to thrive when doing mathematics. Acculturation is most naturally achieved through total immersion in the culture; thus, an inquiry-based classroom in which students are actively engaged in mathematical problem-solving and in which they are required to explain and justify their reasoning is the ideal environment in which to acculturate our students as mathematicians.

A Writing Seminar on Mathematical Topics: Changing Views by Considering Perplexing Counterfactual Themes.

In this talk I describe the first-year writing seminar I taught at Cornell University for five years. I designed a course unique in content, goals, means, and expectations, aimed mainly at students with a strong background in mathematics (future majors in mathematics, engineering, computer science, sciences). To achieve my proposed ends, chiefly an overhaul in students' general beliefs about mathematics, I asked the students to read selected writings on mathematics, to consider certain surprising possibilities (including counterfactuals), to listen to professional mathematicians invited as guests, to discuss and react to their peers' opinions, and to examine in new light the knowledge of mathematics they have acquired throughout their school career. The course gave students a sense of self-awareness toward personal aspects, choices, and assumptions involved in gaining mathematical experience. Although I prepared this seminar for (and I taught it to) undergraduates, another good audience for it would be formed by high-school mathematics teachers.

Inquiry-Based Instruction in a Standard Differential Equations Course for Math Education Major.

This study describes a suitable use of inquiry-based mathematics instruction in a standard elementary differential equations course. Students, who are math education majors, are introduced to linear and nonlinear systems of ordinary differential equations at the early stages of the course. Students, who worked in groups, were engaged in developing locally relevant models such as urban/rural interactions, mosquito growth, seasonal hunting and fishing, and others. Computer technology such as Maple was used to analyze the models, and to explore the various possible outcomes as parameters change. Students’ attitude toward the pedagogy used in the course as well as other assessment tools will be shared.
Modified Moore Method is a virtually entirely student-driven approach to learning, and it is generally considered to be an extreme version of inquiry-based learning. I will discuss my experiences in teaching proof via this method, and will focus on the "best practices" I have discovered along the way. In particular, I will note how these practices speak to one of my overarching pedagogical goals: transitioning students' perception of mathematics from that of a "cookbook" to more of a "toolbox". Lastly, I will discuss some course logistics, as well as observed outcomes, both measured and anecdotal.

Mathematics courses are most often described in terms of their mathematical content. But the lessons that students retain forever frequently include attitudes about and habits of inquiry, curiosity, and effective exploration of the unknown. A basic challenge for us as educators is to design class experiences that systematically instill in students effective thinking habits that last a lifetime.

We know it is beneficial to train students to think critically and mathematically during their early mathematical lives. To this end, I teach Intermediate Algebra and College Algebra in an inquiry-based learning style. The students read the textbook before class and formulate questions about the material to be covered in class the next day. During class they solve problems about that topic, and solutions are presented and discussed before the end of the period. After class they then complete standard homework about the topic, and are encouraged to bring any unresolved homework, worksheet or reading questions for the next class period. We will discuss this method of engaging students, focusing on how student questions evolve during the semester, and on student feedback.

First year undergraduate business students, like other college freshmen, usually expect to be told in class what they need to know for the test." They tend to resist posing questions unless it relates to how they do their homework. However, posing and investigating questions is a lifelong learning skill that is particularly important for business professionals. In this talk, we will share attempts to stimulate question-asking and exploration in an inquiry-based quantitative reasoning course for business students. We ran two sections of this course. One section was linked with a freshman writing course and took place in a classroom whose configuration was designed to encourage inquiry. The other section was unlinked in a standard classroom. We will discuss the assignments designed to promote question-asking in both the linked and unlinked sections as well as data comparing the two sections. The data include the frequency and quality of questions posed in each class and results of a math anxiety and math beliefs survey administered to both sections at the beginning and end of the course.
Homework Presentations in Calculus I.

With an eye toward improving technical communication and shifting (perceived and actual) mathematical authority to students, homework presentations have been employed in a Calculus I course over several semesters. Presentations are student led, whole class discussions that occur once a week; class consensus determines when a problem is finished. Informal evidence suggests a variety of student benefits, including improved written and oral communication skills and increased engagement with homework. This talk will present rationale for student presentations, tips for implementation, and preliminary outcomes related to student attitudes and skills.

Teaching Inquiry through Calculus TACTivities.

Calculus is often thought to be a course where students learn a specific skill set. Limits, derivatives, and integrals are all topics that are covered and hopefully learned in a standard calculus course. It is not often in a calculus course that one thinks about learning how to ask good mathematical questions and explore these questions. In this session, we will share activities that are tactile, hands-on, and engage the students while at the same time teaching them the basic skills of calculus. The activities being created for the calculus series are designed to teach students to explore mathematical ideas without a teacher telling them what to do. The results of the piloting of these activities at two universities will also be shared.

Raising Calculus to the Surface: Discovering geometric connections behind multivariable calculus.

The multivariable setting introduces extensions to many major themes of single variable calculus. The algebraic conditions necessary for these new theorems are often quite subtle but have beautiful geometric interpretations. We have developed physical surfaces, measurement tools, and activities which help students discover potential connections across the multivariable calculus curriculum. We will share how these materials help groups of students formulate and share their own conjectures with the class as well as test the conjectures of their peers before formal lecture. In this talk, we will demonstrate how these materials let students explore the relationship between the geometric features of contour plots and the value of various single, double, and triple integrals. Raising Calculus to the Surface is funded in part by the National Science Foundation.
Discovering the Art of Inquiry: Creating a Culture of Asking Open Questions.

What is needed to support students in cultivating lifelong habits of curiosity? We claim that students will naturally start asking interesting questions if the majority of what they see or hear in their class are questions: course materials dominated by questions rather than facts, a teacher who answers questions with questions, and a classroom where questions focus the day to day work of the group. In this "talk", participants will explore, share, and inquire into tools and activities that can support a culture of inquiry. Discovering the Art of Teaching IBL (artofmathematics.org/classroom/) is a growing collection of pedagogical tools we are learning about for creating a culture of inquiry in our mathematics classes. Discovering the Art of Mathematics (artofmathematics.org) is an NSF-funded project to develop inquiry-based course materials and to provide pedagogical resources to make inquiry-based learning come alive in mathematics courses for liberal arts majors.

Nurturing Inquiry in a Moore Method Geometry Classroom.

A (Modified) Moore Method classroom is normally viewed as a place where students learn to answer questions rigorously, but it also serves as a good platform for teaching students how to ask questions the way a mathematician does. We will discuss several practical ways in which this can be done, illustrated by actual classroom events in the author's Euclidean Geometry course. We shall discuss how careful construction of a class task sequence, a few "presentation management" techniques, and a healthy respect for students can be combined to welcome students into the community of mathematical exploration.

Ask questions to encourage questions asked.

Among our many educational goals are that we want students to pose appropriate questions, and we want students to initiate exploration of those questions; indeed, we want inquiry and exploration to become habits of mind. One effective way to inculcate these habits is to embed structured practice of these skills into every class. An instructor can model the asking of questions and the initiation of explorations. S/he can request explicitly that students ask questions and can prompt students to initiate explorations. By having students share questions, an instructor can guide students in evaluating the potential fruitfulness of those questions. An instructor can provide spaces so students can realize on their own that there are questions to be asked, or that exploration is the next step. The implementation of such modeling, requests, guidance, and timing is both nuanced and instructor-specific; discussion of these aspects will be the focus of the talk. Additionally, how structured inquiry-skills practice might be incorporated into a class depends on context. We will give examples of implementing these techniques in an IBL-influenced calculus class, in a partially IBL mathematics for liberal arts class, and in a completely IBL class for gifted high-school students.
Transitioning students from consumers to producers.

In response to assessment reports identifying weaknesses in communication and reasoning of junior and senior mathematics majors, we have developed a 3-credit semester-long course that is required for all first-year mathematics majors. The focus of this course is on reasoning and communication through problem solving and written mathematical arguments in order to provide students with more experience and training early in their university studies. The goal is for the students to work on interesting yet challenging multi-step problems that require almost zero background knowledge. The hope is that students will develop (or at least move in the direction of) the habits of mind of a mathematician. The problem solving of the type in the course is a fundamental component of mathematics that receives little focused attention elsewhere in our program. The course will be taught via an inquiry-based learning (IBL) approach with an explicit focus on students asking questions and developing conjectures. In this talk, we will describe the structure of the course and our plan for transitioning students from "consumers" of mathematics to "producers".

How Students Experience a Mathematics Program with an Inquiry-Based Philosophy.

The mathematics department at Wheelock College has a strong focus on teacher education, including a mathematics major for prospective preK-8 teachers. The department has a commitment to inquiry-based pedagogy within all our courses, as we know that teachers' default is to teach as they were taught, and most of our students had little previous experience with inquiry-based learning before coming to Wheelock. There are many resources to support inquiry-based pedagogy in introductory courses for prospective teachers, but implementing more advanced inquiry-based courses for this population is less charted territory and hence more challenging. This talk will discuss a small qualitative study of students' struggles and advances through a major committed to inquiry-based pedagogy, with an emphasis on students' experiences in courses such as Algebra & Number Theory, Geometry, and Probability & Statistics.

Methods for Democratizing Inquiry for K-16 Students and Teachers.

After designing and implementing a professional development workshop for middle and high school teachers focused on incorporating Inquiry-Based Learning (IBL) into their teaching, we found that despite their good faith attempts to implement traditional methods of IBL, inquiry did not readily manifest itself in their classrooms. Consequently, we endeavored to unearth inquiry’s theoretical foundations and their implications for teaching in order to promote an inquiry orientation and a more equitable mindset that democratizes access to authentic mathematical activity by honoring the diversity of students’ mathematical knowledge. We believe such an approach not only allows students to realize the benefits of an inquiry orientation for learning mathematics, but also develops their ability to understand and possibly influence "real" problems that exist outside the classroom and in their communities. In this talk we will emphasize the significance of teachers' and students' beliefs about mathematics and about learning. We will also share examples of activities that emphasize the centrality of mathematical tasks in helping teachers cultivate an environment of inquiry and equitable access.
Faculty Knowledge of Teaching in Inquiry-Based Learning Mathematics.

In the context of using Inquiry-Based Learning (IBL) for teaching, over 70 instructors teaching a wide range of university mathematics courses and with various levels of familiarity with IBL filled out bi-weekly logs about the challenges they had teaching with these strategies and the solutions they had found. The analysis of these pairs of concerns and solutions expressed over the three-year study reveals, unsurprisingly, that faculty draw from different domains of teacher knowledge for teaching to solve concerns that arise as IBL is implemented in their classrooms. We see, however, differences depending on the type of work the teacher needs to do, and a weak link due to their level of familiarity with IBL. A further specification of the nature of this knowledge in the different areas in which teaching is manifested is needed.

Critical Components of Inquiry-Oriented Teaching.

Over the last decade, undergraduate mathematics researchers and curriculum developers have generated inquiry-oriented curriculum materials for courses from calculus through abstract algebra. Here, we take inquiry-oriented to describe both the student and the teacher's role in the classroom. Students “learn new mathematics through inquiry by engaging in mathematical discussions, posing and following up on conjectures, explaining and justifying their thinking, and solving novel problems”, while “teachers routinely inquire into their students' mathematical thinking and reasoning” (Rasmussen & Kwon, 2007, p. 190). The work represents our efforts to define and map the domains of inquiry-oriented teaching. Specifically, by drawing on empirical and theoretical research on inquiry-oriented teaching, and by analyzing classroom video data, we will present a list of critical components for inquiry-oriented teaching. These critical components will be illustrated and exemplified by analyzing inquiry-oriented classroom video data.


Math courses for future teachers are arguably the most important math courses we teach. Teaching the full spectrum of inquiry is especially important for those entering the teaching profession. The current challenge is to provide prospective teachers, who may have never experienced inquiry, opportunities to experience inquiry, to understand the merits of inquiry, and to prepare them so that they could one day teach via inquiry. Such lofty goals are difficult to achieve, but are worthwhile to pursue. In this talk, the design and implementation of a capstone course for future secondary school math teachers are discussed that attempts to start to address the described challenges above.
SIGMAA on the Philosophy of Mathematics Guest Lecture
Mathematical authority and inquiry-based learning.

This talk will describe Inquiry-Based Learning (IBL) and its interaction with notions of mathematical authority. The talk will begin with a brief history of IBL, from Moore to current practice, and include a working definition of IBL. The talk will also explore the interplay between IBL and notions of mathematical authority, centering on the questions of, When is a proof considered valid? and, Who can validate a proof? There will also be a discussion of the research on students’ views of mathematical authority and the impact of these on student achievement.

Tuesday January 13, 2015

8:00 a.m. Samuel Cook*, University of Alaska Anchorage 204A, Convention Ctr
Nicole Seaders, Oregon State University Room

To Each Their Own: A Semester Project Emphasizing Continuous Conceptual Involvement and Inquiry.

The project described below arose from an attempt to find a way to give students some ownership over their learning, involve them at every step, and infuse them with the joy and challenge of discovery and research. Students are assigned individual objects to study throughout the term, using the techniques presented in class as the tools of their discovery. Students learn to ask and explore mathematical questions about their individual object and investigate questions of personal interest. Solutions to problems encountered in initial trials of the project are discussed.

8:20 a.m. Jennifer Nordstrom*, Linfield College

Using Game Theory to Foster Inquiry and Writing.

Linfield College requires an Inquiry Seminar of all first-year students. The purpose of this course is to introduce students to academic discourse through writing. As the course title suggests, it motivates writing as a means of posing interesting questions and providing reasoned arguments to answer these questions. Faculty members from all disciplines teach Inquiry Seminars, focusing on themes they find particularly interesting. Although undergraduate mathematics, as a discipline, may seem miles away from undergraduate writing, especially in first-year courses, this talk will focus on the success of a first-year writing course with the theme of mathematical game theory. Mathematics provides a robust framework for students to explore questions of rationality, value, cooperation, and societal versus individual good. Such questions can engage students with wide-ranging interests and backgrounds in mathematics. We will discuss how educational approaches familiar to mathematicians coincide with those of writing faculty, and how these pedagogical methods can be used to encourage students to develop habits of inquiry that can apply to all disciplines.
8:40 a.m.  **Elin Farnell*, Kenyon College**  

*Puzzle Pedagogy: Riddles and Their Value in Mathematics Education.*  

Logic puzzles and riddles have long been sources of amusement for mathematicians and the general public, alike. In recent history, they have taken a prominent role as standard interview questions, especially within the technology industry. I propose that puzzles can serve a useful role in a classroom setting as a basis for discussion of the nature of mathematics, for development of problem-solving skills, and as a means of engaging students from a broad range of mathematical backgrounds. In particular, I suggest that puzzles be used as a tool for encouraging curiosity, creativity, and persistence, both within the classroom setting and in the practice of mathematics more generally. In this talk, I will present a collection of puzzles and discuss underlying concepts that contribute to the development of students' mathematical inquiry.

9:00 a.m.  **Peter Lawson Maceli*, Columbia University**  

*Graph theory by example.*

For the past two years, I have been teaching a class in graph theory for high school students as part of the Columbia University Science Honors Program. Since graph theory is such an accessible and visual field of mathematics, it provides an ideal setting for a class where the main focus is on developing mathematical intuition and instinct. Questions, problem solving, and student examples drive the course, which is structured more as a guided exploration rather than lecture. Many deep concepts and ideas in graph theory are illustrated by small graph examples. Allowing students the freedom to make these discoveries themselves helps them develop a new perspective on the roles creativity and ingenuity play in the mathematical process. In this talk, we describe several lessons from the course, which explore some non-traditional topics from graph theory.

9:40 a.m.  **Lara Pudwell*, Valparaiso University**  

*Teaching Inquiry through Experimental Mathematics.*

Since 2009, Valparaiso University has offered a junior-level course in Experimental Mathematics as an elective for mathematics majors and minors. Rather than focusing on a particular list of content, the goal of this course is to cultivate an attitude of exploration in students' approaches to new problems. Along the way students learn basic programming using a computer algebra system and work on independent research projects throughout the semester. This talk will describe the setup of the course and give particular examples of experimental class activities.

10:00 a.m.  **Diana White*, University of Colorado Denver**  

*Using a Non-Traditional Mathematical Operation to Teach Inquiry.*

Problem posing and mathematical exploration are key disciplinary components of mathematics, yet they are often overshadowed by the push to cover content. In this talk, we discuss an open-ended mathematical problem that has been used repeatedly by a variety of mathematicians in both undergraduate mathematics courses and professional development workshops for teachers. We provide an overview of the problem, discuss how it leads naturally to problem posing, mathematical exploration, conjecturing, communication, and level-appropriate rigorous mathematical argumentation. We also point out how it crosses into a variety of mathematical content areas and connects up with ongoing mathematical research. Finally, we provide a brief qualitative overview of survey results from students regarding the impact of these types of problems on their perspective of mathematics.
Extending mathematical problems.

As mathematicians, we often create new problems by extending our recent work: considering further cases, generalizing, or exploring a new direction uncovered by solutions to previous problems. In this talk, we will discuss two assignments that ask students to pose their own problems by extending their work on assigned problems. One assignment is from a discrete mathematics course typically taken by sophomores and one is from the capstone course for the mathematics major. We will share the assignments, how students have performed and comments they have made, and compare how the expectations and performance varies between the courses.

What do you notice? Using conjecturing activities to teach inquiry and ignite student's curiosity about mathematics.

What happens next? Does it always work? What if we tried this instead? How many are there? These sorts of questions are part of any mathematician's toolkit. We are well trained in the skill and art of inquiry. But how do we ignite students' curiosity and help them develop this ability? For the past fifteen years I have taught a sophomore level discrete mathematics course that teaches inquiry through conjecturing activities. After all, conjecturing is at the core of what mathematicians do { it is our research experiment, our way of thinking through an abstract question, and prelude to developing a theory. Teaching conjecturing helps students grow their inner mathematicians, preparing them for advanced courses and undergraduate research. But the benefits extend beyond. Conjecturing activities ignite student's curiosity and develop their ability to inquisitively explore new ideas. As an added bonus, students enjoy making conjectures and so they spend a lot of time working on classwork in and out of class and they are more likely to persist in their study of mathematics. In this talk I'll describe why we teach conjecturing and illustrate several successful mathematical conjecturing activities and reflections on what makes these activities work.

Engaged Calculus - Building Community-Centered Inquiry into a First Semester Calculus Course.

We describe the implementation of Engaged Calculus at the University of Texas at Arlington. In this section of Calculus I, populated by primarily _rst-time, full-time freshmen, students developed and explored research questions that they could investigate utilizing the mathematical tools of calculus which they were simultaneously learning in the course. Their research questions were all based in issues they chose related to their university community, thus requiring them to become more fully engaged as UT Arlington Mavericks { for example, one group modeled the laundry machine availability in the dorms on campus to predict the best times to do laundry. These research projects, completed in groups of 4 or 5 students, spanned the entire semester, and culminated with written reports and poster presentations. We describe both the structure of the course including the timeline for the projects, the supports that were provided as the students collaboratively developed their research questions, and how this focus on mathematical inquiry influenced the student outcomes in terms of their calculus skills and understanding, and their attitudes about how real-world problems can be better understood using mathematics.
Distinguishing Mathematical Definition by Doing the Coochy Coo.

Most students believe that definitions play an important role in the learning of mathematics. However, they struggle with transitioning from performing procedures to reasoning from definitions as they move into more sophisticated mathematics. Further, students tend to rely primarily on their concept image and intuition about a concept, especially visual/mental images, instead of the concept definition. The issue of limited concept images and the lack of a strong connection between a concept image and its mathematical definition need to be addressed at all levels of study in undergraduate and secondary mathematics. The goal of this session is to present an activity designed to help students focus on the concept definition while isolating their natural inclination to rely solely on their concept image when working on a mathematical task.

Definition Construction and Developing Mathematical Inquiry.

In this session, we will present a small unit designed to encourage students to define, conjecture and (dis)prove their own 'kind' of numbers and evidence of student success (and struggles) with the unit. Although the students were preservice middle school math teachers, the unit could be used in other courses at or beyond a college algebra level. After introducing some properties of integers such as commutativity and closure, as well as the 2k+1 and 2k forms of odds and evens, students were asked to construct their own 'kind' of number (e.g., Apple Numbers are of the form 3n+7 where n is an integer). Students then conjectured about their numbers and (dis)proved their conjectures. Common conjectures involved closure and whether the 'kind' of number was a multiple of some factor. Minimal instruction related to proving was included because this group had no formal proof background. After engaging in their own definition construction for several classes, the students began to conjecture more advanced material like the twin prime conjecture. Overall, the results suggested that they struggled to prove their conjectures but were able to develop many conjectures and seemed to adopt a shift in their thinking towards questions of the sort, "What if" and "Maybe it's true that."

Experiments in Conjecturing.

Our department requires students to take a seminar on a topic not covered in standard mathematics courses. I chose to discuss Continued Fractions since it sounded like something the students would find interesting and I knew little about the topic. At the first meeting, I gave the basic definitions and did a few examples. After each session, there was a three part assignment. Part 1 consisted of different types of computational problems. Part 2 was always the same: Make at least one conjecture based on the computations from Part 1. Part 3 asked them to prove or disprove at least one of the previous conjectures. During the class discussion, students were expected to argue for their conjectures (as opposed to proving them). At the start, conjectures tended to be sweeping ("All . . ."). Students needed help with the idea of hypotheses. Sometimes this was done during the class discussion, or someone would give a counterexample that narrowed the scope of a conjecture. By the end of the term, they got the idea of stating conditions. The students spent a lot of time discussing the material, both in and out of class. I feel that the seminar was a successful experiment in giving the students a taste for creating mathematics on their own.
TRIGONometry : An Inquiry of Triangle Measurement.

Often students arrive in a college level pre-calculus class with varied backgrounds in geometrical thinking. Some states' school curricula specify an algebraic treatment to high school geometry and recent textbooks relegate constructions with a compass and straight-edge to enrichment activities instead of the fundamental exercises. So, trigonometry, measuring triangles, has become essentially an abstraction to students today instead of the practical skill that it has been through the millennia. This presentation will focus on activities developed to enhance students' trigonometrical thinking through inquiry and practical applications.

Using Games as a Invitation for Inquiry.

Students are most invested in answering questions that they pose themselves; However, posing questions is a skill that does not come naturally to most students and is not universally encouraged in classrooms. In order to get students to pursue deep questions, you need to both provide a setting which sparks curiosity and an environment which values pursuit of the answers. In this talk I will outline how I used the board game Ticket to Ride to inspire the direction of a discrete structures class. At the beginning of the semester students learned how to play the game (which takes places on a board which is an edge-colored graph representing potential train connections between cities in the United States.). As a class, they made a list of questions about the game. During the rest of the semester, we learned the mathematics we needed to answer these questions using a set of IBL notes. We selected topics according to their usefulness towards the initial list of questions. Our list inspired the investigation of spanning trees, Steiner trees and shortest path algorithms. The class found an innovative strategy for winning the game.

Using Games to Engage Students in Inquiry.

The success of inquiry-based learning courses rests upon the engagement of the students; they have to want to understand in order to work with the material at the level that will promote understanding. This talk will focus on sharing the mathematics of games as discovered in an inquiry-based course at a small, liberal arts college. In the course, students formed conjectures and sought resolution as individuals with the support of the instructor and the classroom community. The mathematics addressed will include modular arithmetic, geometry, binary operations, and graph theory. The presenter will share ideas on how to get students to engage with these mathematical topics through inquiry in the context of exploring several well-known games.

Apply inquiry-based mathematical teaching in actuarial science classes.

Inquiry-based learning has been widely used as a teaching and learning tool in developing students' critical thinking skills. While mathematics is perceived as one of the most abstract subjects, actuarial science is an excellent outlet for applying mathematical theories to exploring and solving real-life problems. Therefore, the inquiry-based mathematical teaching proves to be a really important part of these courses. In this paper, the author will use some examples in core actuarial courses, such as Probability and Financial Mathematics, to illustrate the methods of teaching students to ask and explore mathematical questions in actuarial science classes.