the 3rd Annual
Legacy of R.L. Moore Conference

Modified Moore Methods, As Used Today

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The Lila B. Etter Alumni Center
The University of Texas at Austin
Co-Chaired by

Ben Fitzpatrick
Auburn University

&

John W. Neuberger
University of North Texas

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Q: An Invitation to Invention

Howard Cook
University of Houston (Professor Emeritus)

In his Math 624 classes, R.L. Moore asked his Students about the existence of a certain collection Q of point sets. I will talk about the question, my perception of its purpose, and its value to the development of the students in the class.
Moore-Method in the Math-for-People-Who-Don’t-Want-To-Take-Math Course

G. Edgar Parker
James Madison University

Many colleges have courses in their mathematics offerings which owe their existence to giving the least mathematically-prepared or the least mathematically-motivated students at the college a fighting chance of fulfilling a mathematics requirement without entering the track that contains the calculus sequence. The author has been involved in teaching such a course since 1978, first at Pan American University (now UT-Pan American), and since 1985, at James Madison University. In this talk, he will discuss how he uses Moore-method in such a course.

The talk will address three pedagogical issues:

i. preparing the students to take the “Moore plunge”,

ii. actual conduct of the Moore unit, and

iii. evaluating and using the outcomes.

Three different problem sets the author uses for his Moore-style unit will be used to focus the discussion.
On Three Crucial Elements of Texas-Style Teaching as Shown to be Successful in the Secondary Mathematics Classroom

Lee Mahavier
Meadowcreek High School

The author discusses three elements integral to Texas-style teaching and gives a rare firsthand look at Moore Method use in the high school classroom. These elements, which relate to classroom atmosphere and rapport with students, were employed by Professors Ettlinger, Moore, and Wall, and should be emphasized as the method gains more widespread appeal. The author gives examples of three generations' successful use of these crucial techniques, which transcend factors such as student age and course level.

One of the elements, respect for learning, is lacking in the minds of many high school students today. The author illustrates how to add this to the curriculum by means of classroom rules, encouraging self-reliance, and the teacher's example. Quotes from high school students show how they have consequently been inspired to think for themselves, appreciate their own minds, and enjoy discovering mathematics.
R.L. Moore's methods are known to have been very effective with certain classes of students. Part of the success of his method lies with the independent thinking that Moore required of his students, and part of it surely owes to the immersion his techniques required of students.

I will discuss in a little detail a recent and very substantial foray into immersion and discovery learning, the project MASS at Penn State. This program is motivated by the Hungarian semesters which took place in Budapest in earlier years. It involves students spending a semester at Penn State working entirely on mathematics. I will describe both the curricular arrangements and the environment that has been created for this program.
Why Are Moore Method Courses Effective?

Annie Selden (presenting)
Tennessee Technological University
Arizona State University (Visiting Professor)

John Selden
Mathematics Education Resources Company (President)

Having taught a variety of Moore Method mathematics courses – topology, algebraic topology, abstract algebra, geometry, topological semigroups – over many years, we have often wondered why they worked. Is there a theory, or a mechanism, that might explain their effectiveness? The best of these courses seemed not only to teach students mathematics, but also about proving theorems, about themselves, and about what we might call the culture of mathematics.

When behaviorist psychology was in its heyday, one of us viewed this in terms of reinforcement. When a student found a proof and it was accepted, this amounted to a very positive reward. Having to admit that one did not have a proof, or presenting a proof with a mistake in it, was negative. The way conditioning works suggests, for example, emphasizing the positive and minimizing the negative experiences, even if negative experience might serve as a source of motivation.

While such analyses were of some use, behaviorist psychology limited itself to analyzing externally observable events. Consideration of what goes on inside the mind is more promising and no longer taboo. There are now more cognitively-based ways of looking at the teaching/learning of mathematics and a constructivist perspective may be helpful. For example, Vygotsky's Zone of Proximal Development (ZPD)
may have some explanatory power relative to the Moore Method. Vygotsky was a contemporary of Piaget and the ZPD refers to what a student can do in the presence of a "teacher" but not alone. We will discuss how this and a number of other ideas from research in mathematics education might help to understand Moore Method teaching and the teaching of "transition" courses to mid-level undergraduate mathematics students.
Ask “Why?”, Insist on Seeing, Experience Deeply, Learn from Others

David W. Henderson
Cornell University

I am a second generation Moore descendant (a student of R.H. Bing). My learning under the Moore Method has greatly influenced my teaching and life over the years. I would like to emphasize what I think are the four most important aspects of my teaching. I think that each of these four is something I learned from my experiences in and with the Moore Method – even though they might not be in everyone's experience of the Moore Method. These four aspects apply not just to learning mathematics but also to teaching mathematics, and, in addition, they apply to life far beyond mathematics.

1. Always be questioning, especially **ASK: WHY?** (Do not believe something just because an authority says it.)

2. **INSIST ON SEEING** (understanding) why something is true.

3. Look to your deep **EXPERIENCE** for answers.

4. Be open to **LEARN FROM OTHERS** – teachers from students, students from students, and students from teachers.

I will illustrate with examples from my learning and teaching.
At colleges and universities of all kinds, mathematics departments are re-thinking their missions and seeking to integrate teaching, research and scholarship, and service. Project NExT (New Experiences in Teaching) is a professional development program that assists new and recent Ph.D.s in the mathematical sciences as they assume the diverse professional roles that they are expected to play. Each year, about sixty new faculty participate in a series of workshops and other activities that address a broad range of issues, focusing on the teaching and learning of undergraduate mathematics. Many of the teaching strategies to which the participants are introduced incorporate a significant component of discovery learning. During the last six years, over 400 new faculty have participated in this program, which is sponsored by the Mathematical Association of America, with major funding from the Exxon Mobil Foundation.
The R. L. Moore Method at the Secondary Level

David McRae (presenting)
Woodberry Forest School

H.W. Straley
Woodberry Forest School (Emeritus Chair of Mathematics)
Babson College (Visiting Professor)

The R. L. Moore method, with some modifications, can be very effective at the secondary level. The authors will describe a program that has, for over thirty years, effectively helped students learn how to both problem solve and do mathematics. This program has continued to be successful even with the retirement of the original instructor. The authors will describe the school setting, the student population, the mathematical content, the R. L. Moore related pedagogy.
Adapting Moore's Method to the Distance Education Classroom

Dale Daniel  
Lamar University

Wm. Ted Mahavier  
Nicholls State University

Craig Pember  
Educational Technology Center for Distance Learning and Professional Development at Lamar University

The distance education classroom presents special challenges to the teacher who wishes to employ Dr. Moore's method. This is particularly true for lower to intermediate level mathematics courses delivered by live two-way interactive video to one or more remote sites.

D. Daniel and C. Pember will address:
(1) a brief discussion of the SETTEN (Southeast Texas Telecommunications Education Network),
(2) the precise setting in which they have delivered such courses,
(3) their experiences in modifying Dr. Moore's method in such settings, and
(4) offer suggestions for techniques that they have found helpful for successfully doing so.

Wm. Ted Mahavier will address the current state of the materials he and James P. Ochoa have collected and the potential formats in which they can be offered to faculty wishing to implement the method.
Adapting the Moore Method to a Virtual University Course

D. Reginald Traylor
University of the Incarnate Word

The Moore Method of teaching is a proven method in traditional classrooms. Success of modifications of the Moore Method varies from one setting to another and is much influenced by the personality of the instructor. Those efforts that are most successful seem to hold fast to three principal characteristics: 1) homogeneous background of the students; 2) no reading by the student nor discussion allowed between the students; and 3) student peer evaluations of other student’s presentations of efforts of proof. New technology provides opportunities to extend the Moore Method to Virtual University courses. This paper describes a proposed Moore Method topology course, based on R.L. Moore’s Axiom 0 and Axiom 1 (3), that will be delivered in the summer and fall of 2000 by way of Virtual University technology. Items 1), 2) and 3) above, and necessary modifications of those, will be discussed in the framework of the proposed Virtual University delivery of the course.
My Experiences Using Modified Moore Methods in Undergraduate Courses

Melvyn Jeter
Illinois Wesleyan University

This paper reviews my experiences teaching several undergraduate mathematics courses using variations of the Moore Method. Attention will be focused primarily on three courses, Techniques of Proof, Topology, and Topics in Geometry. The first of these is a sophomore course which is taken by our majors in either their freshman or sophomore year. The second two are senior level courses. Topics in Geometry deals with convex sets in Euclidean space. These courses are well enrolled. Our graduating seniors often list them as some of their most meaningful undergraduate experiences.
The Moore Method in the Teaching of Abstract Algebra

M.S. Jagadish
Barry University

The subject of Topology and Abstract Algebra share the fact that they are both abstract and at the same time have many concepts arising from the generalization of well known concepts about numbers and other “concrete” objects. The Moore method in a modified form can enhance the understanding of any student who is willing to work. In addition this method makes the student realize how the axioms of abstract algebraic systems have developed from their classical roots. In the class carefully selected examples and simple deductions are given to the students in worksheets in advance. These problems are selected in such a way that the student works with concrete objects and the necessary abstract ideas are isolated from them. The worksheets are turned in before the concepts are discussed in class. In the teaching of algebra, discussing all the structures and their properties formally is important. The work done by students independently and the formal discussion in class are alternated. Some examples of these activities related to the discussion of Groups, Rings and Fields will be presented.
Teaching Oral and Written Communication For a Modified Moore Method Calculus I Class – And Anything Else

Charles Allen (presenting)
Drury University

Carol Collins
Drury University

Jeanie Allen
Drury University

Peter Renz in the August-September (1999) issue of the Focus states that the essentials of the discovery method are: Motivation, Discovery, and Presentation. We have found that freshmen and sophomores enrolled in a Calculus I course encounter difficulties in both the motivation and presentation aspects of the Moore method.

In this paper we discuss the techniques that we used to incorporate the teaching of both oral and written communication skills into the structure of the class. We also discuss how we approached the difficulty of motivation.

We compared the classes we taught using these techniques and the modified Moore method with calculus classes taught to similar students the same semester but using a reformed approach. We conclude our paper with an assessment of our results.
What Should One Do With AP Calculus Students?

William S. Mahavier
Emory University

Emory University requires that each student take a freshman seminar. The maximum class size is 15 and active student participation is required. For two years I have taught a seminar for freshmen who made a 4 or 5 on the AB Calculus AP exam. They receive a letter saying the course is for students who don't like lectures and enjoy working on math problems. Those brief comments have drawn very good students to the course.

Using my version of the Moore method, I give them definitions and problems and expect them to show their attempts at solutions in class. I will describe the problem sequence in which we develop properties of the natural logarithm function much as H. S. Wall does in his book Creative Mathematics. I will discuss some of their discoveries that were new to me, and how I lead them into the notion of limits and convergence for series and sequences. I will describe how the class is conducted, how I get them to write correctly and my grading system.

The course has been popular enough that I am teaching it for a 3rd time in the spring of 2000. This time the class consists of students who did not do as well on the AP exams and took our regular calculus in the fall but were bored.
The Confessions of a Heretic

Carol Schumacher
Kenyon College

One of the prominent features of R.L. Moore's pedagogy was his ban on collaboration. At Kenyon, we do not strictly observe this ban – in fact, we sometimes find that collaboration can further the goals of a Moore-style course: it can actually foster mathematical self-sufficiency in our students.

I will try to resolve this apparent paradox by explaining how I use small group collaborations in my "introduction to proofs" course and its role in the formation of young mathematicians.
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Did it never occur to you that when you read or listen to a proof of a theorem that you had never heard of till someone stated it, ... you are thereby acquiring information (of a sort) but depriving yourself of the opportunity to work it out for yourself and thereby, perhaps, to develop that much more power instead of just acquiring that much more information?

What does information amount to compared to power?

(excerpt from Professor R.L. Moore’s Letter to Mary-Elizabeth Hamstrom, 1948)