On the research of Mary Ellen Rudin

Mary Ellen Rudin was one of the leading topologists of our time. Besides solving a number of well-known outstanding open problems, she was a pioneer in the use of set-theoretic tools. She was one of the first to apply the independence methods of Cohen and others to produce independence results in topology.

Mary Ellen got her PhD in 1949 at the University of Texas, Austin under the supervision of R.L.Moore. There is an excellent biographical interview of her published in:

> Albers, Donald J. and Reid, Constance, *An Interview with Mary Ellen Rudin*, College Math. J. 19 (1988), no. 2, 114-137.
>
> `http://www.jstor.org/stable/pdfplus/2686174.pdf`

An entire volume devoted to articles describing her mathematics appears in the Annals of the New York Academy of Sciences. The quotations below are from it.

> *The Work of Mary Ellen Rudin*, Papers from the Summer Conference on General Topology and Applications in honor of Mary Ellen Rudin held in Madison, Wisconsin, June 26-29, 1991. Annals of the New York Academy of Sciences, 705, 1993.

In her thesis she gave an example of a nonseparable Moore space that satisfies the countable chain condition. She published the results in three papers in the Duke Math Journal. To quote Steve Watson,

"This cycle represents one of the greatest accomplishments in set-theoretic topology. However the mathematics in these papers is of such depth that, even forty years latter, they remain impenetrable to all but the most diligent and patient readers."

In 1955 she used a Souslin tree to construct a Dowker space. A Dowker space is a normal space whose product with the unit interval is not normal. The existence of a Souslin tree is consistent with ZFC, but not provable from ZFC. In 1971 she constructed a Dowker space without using any extra set-theoretic axiom. The space she constructed has cardinality $(\aleph_\omega)^{\aleph_0}$. It led to an invited address at the International Congress of Mathematicians in 1974. It also led to her interest in the box topology. In 1972 she proved that the Continuum Hypothesis (CH) implies that the box product of countably many compact metric spaces is normal. In 1989, assuming CH, she constructed a Dowker space of cardinality $\aleph_1$.

Mary Ellen is well-known for her work on $\beta\mathbb{N}$, the space of ultrafilters on the natural numbers, starting in 1966. She was co-inventor of two well-known partial orders on this space, the Rudin-Keisler order and the Rudin-Frolik order. The Rudin-Frolik order led to the first proof in ZFC that the space $\beta\mathbb{N} \setminus \mathbb{N}$ is not homogeneous. Under CH, this was already known by a result of Walter Rudin.

She worked extensively on the question of S and L spaces. Mary Ellen produced the first S-space (a hereditarily separable space that is not hereditarily Lindelöf) assuming the existence of Souslin tree in 1972. Her 1985 monograph, *Lectures in Set-theoretic Topology*, devotes an entire chapter to S and L spaces. To quote Stevo Todorčević,

"The terms S and L space, which are predominant in most of the literature on this subject, are first found in these lectures. This shows a great influence not only of this monograph but also of M.E.Rudin's personality on the generation of mathematician's working in this area, since it is rather unusual in mathematics to talk about certain statements in terms of their counterexamples."

In 1999, almost a decade after her retirement, Mary Ellen settled a long-standing conjecture in set theoretic topology by showing that every monotonically normal compact space is the continuous image of linearly ordered compact space. This paper was the final one in a series of five which gradually settled more and more special cases of the final result. The construction of the linearly ordered compact space is extraordinarily complex. To quote Frank Tall,

"Mary Ellen's fame is largely as a producer of weird and wonderful topological spaces – examples and counterexamples. One often wonders how on earth she was able to construct them. She has an uncanny ability to start off with a space that has some of the properties she wants, and then push it and pull it until she gets exactly what she wants. Mary Ellen cheerfully tells people not to read her papers, but rather the later ones by people who simplify what she has done, but she of course is the one who did it first."

Mary Ellen was by consensus a dominant figure in general topology. Her results are difficult, deep, original, and important. The connections she found between topology and logic attracted many set theorists and logicians to topology. The best general topologists and set theorists in the world passed regularly through Madison to visit her and work with her and her students and colleagues.

She had sixteen PhD students, many of whom went on to have sterling careers of their own. To quote Michael Starbird,

"From the perspective of a graduate student and collaborator, her most remarkable feature is the flood of ideas that is constantly bursting from her. $\cdots$ It is easy to use the Mary Ellen Rudin model to become a great advisor. The first step is to have an endless number of ideas. Then merely give them totally generously to your students to develop and learn from. It is really quite simple. For Mary Ellen Rudin."