Let’s all concentrate on T28, T29, T30 this week.

As soon as these are resolved, I want to concentrate on the following:

**Definition 1** The statement that \( q_1, q_2, q_3, \ldots \) is a subsequence of \( p_1, p_2, p_3, \ldots \) means that there is an increasing sequence of natural numbers, \( n_1, n_2, n_3, \ldots \) such that for each natural number \( i \), we have \( p_{n_i} = q_i \).

**Example:** Suppose \( p_1, p_2, p_3, \ldots \) is a sequence and \( n \) is a function with domain the natural numbers so that \( n_1 = 2, n_2 = 4, n_3 = 6, \ldots \) so that \( n(k) = 2k \) for \( k = 1, 2, 3, \ldots \). Then the sequence \( q_1 = p_2, q_2 = p_4, q_3 = p_6, \ldots \) is a subsequence of \( p_1, p_2, p_3, \ldots \). We will use the notation \( (p_n) \) for the sequence \( p_1, p_2, p_3, \ldots \) and \( (p_{n_k}) \) for the subsequence of \( p \) defined by the sequence \( n_1, n_2, n_3, \ldots \). Notice that for any sequence \( n \) defining a subsequence, \( n_k \geq k \) because \( n \) is an increasing sequence.

**Problem 1 Subsequence 0** Prove that the subsequence in the above example converges.

**Problem 2 Subsequence 1 (Kimberly’s Question)** If \( q_1, q_2, q_3, \ldots \) is a subsequence of \( p_1, p_2, p_3, \ldots \) and \( p_1, p_2, p_3, \ldots \) converges to some number \( x \), then must \( q_1, q_2, q_3, \ldots \) converge to \( x \)?

Now that Kimberly’s Question is resolved, we have the theorem:

**Theorem 1** If \( q_1, q_2, q_3, \ldots \) is a subsequence of \( p_1, p_2, p_3, \ldots \) and \( p_1, p_2, p_3, \ldots \) converges to some number \( x \), then \( q_1, q_2, q_3, \ldots \) also converges to \( x \).

**Problem 3 Subsequence 2** Suppose that \( q_1, q_2, q_3, \ldots \) is a subsequence of \( p_1, p_2, p_3, \ldots \) and there is a number \( x \) so that \( q_1, q_2, q_3, \ldots \) converges to \( x \). Is it true that \( p_1, p_2, p_3, \ldots \) converges to \( x \)?

**Problem 4 Subsequence 3** Suppose that \( (p_n)_{n=1}^{\infty} \) is a sequence of points in the closed interval \([a, b]\). Is it true that every subsequence of \( (p_n)_{n=1}^{\infty} \) converges to some point in \([a, b]\)?

Once these are resolved, T31, T32, T33