1. I think we know what we need to prove to resolve T28 and T30. Don’t give up on these, but don’t spend all your time on them either. I created the sequence and subsequence stuff to give us some more basic problems than T28, T30-T33 which are really tough problems worthy of your time and your intellect, but also can be frustrating!

2. The same goes for P31, Weston, don’t give up, but don’t spend all your time on it.

3. Attempt to solve the outstanding sequence problems: SS0, SS3, SS4 and C1.

4. Once we resolve SS0, SS3, SS4 I’ll let us move forward to more integration stuff!

**Definition 1** The statement that \( q_1, q_2, q_3, \ldots \) is a subsequence of \( p_1, p_2, p_3, \ldots \) means that there is an increasing sequence of natural numbers, \( n_1, n_2, n_3, \ldots \) such that for each natural number \( i \), we have \( p_{n_i} = q_i \).

**Example:** Suppose \( p_1, p_2, p_3, \ldots \) is a sequence and \( n \) is a function with domain the natural numbers so that \( n_1 = 2, n_2 = 4, n_3 = 6, \ldots \) so that \( n(k) = 2k \) for \( k = 1, 2, 3, \ldots \). Then the sequence \( n \) defines the sequence \( q_1 = p_2, q_2 = p_4, q_3 = p_6, \ldots \) which is a subsequence of \( p_1, p_2, p_3, \ldots \). We will use the notation \( (p_n) \) for the sequence \( p_1, p_2, p_3, \ldots \) and \( (p_{n_k}) \) for the subsequence of \( p \) defined by the sequence \( n_1, n_2, n_3, \ldots \). Notice that for any sequence \( n \) defining a subsequence, \( n_k \geq k \) because \( n \) is an increasing sequence.

**SS 0** Prove that the subsequence in the above example converges.

**SS 1 (Kimberly’s Question)** If \( q_1, q_2, q_3, \ldots \) is a subsequence of \( p_1, p_2, p_3, \ldots \) and \( p_1, p_2, p_3, \ldots \) converges to some number \( x \), then must \( q_1, q_2, q_3, \ldots \) converge to \( x \)?

Now that Kimberly’s Question is resolved by Tre, we have the theorem:

**Theorem 1** If \( q_1, q_2, q_3, \ldots \) is a subsequence of \( p_1, p_2, p_3, \ldots \) and \( p_1, p_2, p_3, \ldots \) converges to some number \( x \), then \( q_1, q_2, q_3, \ldots \) also converges to \( x \).

**SS 2 (John)** Suppose that \( q_1, q_2, q_3, \ldots \) is a subsequence of \( p_1, p_2, p_3, \ldots \) and there is a number \( x \) so that \( q_1, q_2, q_3, \ldots \) converges to \( x \). Is it true that \( p_1, p_2, p_3, \ldots \) converges to \( x \)?

**SS 3** Suppose that \( (p_n)_{n=1}^{\infty} \) is a sequence of points in the closed interval \([a, b]\). Is it true that every subsequence of \( (p_n)_{n=1}^{\infty} \) converges to some point in \([a, b]\)?

**Definition 2** The statement that the sequence \( p_1, p_2, p_3, \ldots \) is a Cauchy sequence means that if \( \epsilon \) is a positive number, then there is a positive integer \( N \) such that if \( n \) is a positive integer and \( m \) is a positive integer, \( n \geq N \), and \( m \geq N \), then the distance from \( p_n \) to \( p_m \) is less than \( \epsilon \).

**SS 4** The sequence \( p_1, p_2, p_3, \ldots \) is a Cauchy sequence if and only if it is true that for each positive number \( \epsilon \), there is a positive integer \( N \) such that if \( n \) is a positive integer and \( n \geq N \), then \( |p_n - p_N| < \epsilon \).

**Definition 3** An open cover of a set \( M \) is a collection of open intervals with the property that if \( m \in M \) then \( m \) is in at least one of the open intervals in the collection.

**Definition 4** A set \( M \) is D1-Compact if every open cover of \( M \) has a finite subcover.

**Definition 5** A set \( M \) is D2-Compact if every sequence in \( M \) has a convergent subsequence that converges to some point of \( M \).

**Problem C1** Show that if \( M \) is a set that is D1-compact then it is D2-compact.