Name: ____________________________

- Every numbered problem is worth 4 points.
- Do each problem on a separate piece of paper.
- Do NOT use other theorems or problems in your proofs.
- You may use the ‘usual’ properties of the numbers, the definitions, and the completeness axiom.
- Do all of problems 1 - 3; do one of problems 4 and 5.

1. Language
   (a) Complete: The number \( p \) is the first point to the right of the set \( M \) means...
   (b) Complete: The line \( L \) is tangent to the function \( f \) at \((x, f(x))\) means...
   (c) Negate: Every analysis class is fun and hard.
   (d) Negate: The function \( f \) is continuous at \((x, y)\) if \( x \in D_f \) and if \( S \) is an open interval containing \( f(x) \) then there is an open interval \( T \) containing \( x \) so that for all \( t \in T \cap D_f \) we have \( f(t) \in S \).

2. True or false: (Prove or give a counter example.)
   (a) If \( M \) is a point set and \( p \) is a limit point of \( M \) and \( q \) is a limit point of \( M \) then \( p = q \).
   (b) If \( M \) and \( N \) are sets with \( M \subset N \) and \( p \) is a limit point of \( M \) then \( p \) is a limit point of \( N \).
   (c) If \( p_1, p_2, p_3, \ldots \) is a decreasing sequence with range a subset of the positive numbers, then \( p_1, p_2, p_3, \ldots \) converges to some point, \( p \).
   (d) If \( M \) is a set and \( p > m \) for all \( m \in M \) then \( p \) is the first point to the right of \( M \).

3. Continuity: Prove or give a counter example to the statement: If \( f \) is a continuous function on \( M \) and \( x \in M \) is a limit point of \( M \) then \( f(x) \) is a limit point of the range of \( f \).

4. Differentiability: Let \( f \) be the function such that \( f(x) = 3x^2 - 1 \) for every \( x \in \mathbb{R} \) and use your definition above to prove that the derivative of \( f \) at the point \((2, 11)\) is 12.

5. Nested Intervals: Let \( I_n = [-1 - 1/n, 1 + 1/n] \) for every positive integer, \( n \). Prove that \( \{ x : x \in I_k \text{ for all } k = 1, 2, 3, \ldots \} = [-1, 1] \).